Justin Tang
Y10 Trust (23)

Part 1

(a) $L^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

(b) $M^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$M^2 = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

(c) $N^2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$N^2 = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$

(d) $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}^2$

$= \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$= \begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix}$

(e) Yes, Xavi is right

(f) Yes, Xavi's answer is always right, as A equals to $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$, A^2 equals to $\begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix}$.

As Xavi said that $A = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$, then $A^2 = \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$, so susbcribe "$2a^2+2a$" in the equation of A^2 which $2a^2+2a = b$.

So we can see the A^2 is the same as what Xavi says which is $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$.

(g) $LM = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$LM = \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$

(h) $MN = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$MN = \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$

(i) $LN = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$

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(j) $AB = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

$= \begin{pmatrix} 2ab+a+b+1 & 2ab+a+b \\ 2ab+a+b & 2ab+a+b+1 \end{pmatrix}$

let c be $2ab+a+b$

$\therefore \begin{pmatrix} 2ab+a+b+1 & 2ab+a+b \\ 2ab+a+b & 2ab+a+b+1 \end{pmatrix}$

$\therefore = \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(k) $L^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$L^3 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$L^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

(l) $L^4 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$L^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

(m) $L^5 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

(n) $L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ $L^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

$L^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$ $L^5 = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

I noticed that when L multiplied by itself, the 2 x 2 matrices must be corresponding to each other and it proved that |AB| is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

Part 2

(a) $IMT = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$MT = \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$

(b) When I calculate IMT, I am finding the reflection of T.
MT represent the reflection of T in y-axis.

(c) reflection in y-axis

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(d) $M^2 T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^2 \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^2 T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^2 T = \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$ $M^2 T$ is same as matrix T.

(e) $M^3 T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^3 \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^3 T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^3 T = \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$ $M^3 T$ is same as matrix MT.

(f) $M^{-1} = \frac{1}{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

M^{-1} is the inverse of M and it is the same as M.

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(g) $M^{89998}T$

$MT = \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$ $M^2T = \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^3T = \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$ $M^4T = \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^{89998}T$

$= \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$M^{89998}T$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^{89998} T$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} T$

$= \begin{pmatrix} 2 & 5 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

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